New Voting Systems for NY—
Long Lines and High Cost

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As New York decides on new voting systems, one key question is this — how many voters can be served by each voting machine? This number is critical in order to estimate costs as well as to avoid long lines for voters. The New York City Board of Elections recently released a report saying that New York should replace each lever machine by 1 full-face-ballot computer DRE voting machine with voter verified paper trail. Assuming that each voter will take 3.25 minutes to vote, they calculate that 277 voters can vote on each DRE in a 15-hour Election Day. However, the report neglects the effect of non-uniform voter arrivals, DRE outages and extra time needed by voters using special accessibility aids on DREs. We have applied queuing theory, the mathematical study of waiting lines, to carry out computer simulations of realistic elections. We use a scenario with more voters arriving at peak times—early morning, lunch and early evening hours—as is typical during elections. According to our calculations, a ratio of 277 voters per DRE would create unacceptable wait times of 1 hour or longer. Recent elections using DREs have produced extremely long lines in many places around the country, causing would-be voters to leave, thereby disenfranchising them. In order to guarantee reasonably short wait times—even without taking into account DRE outages and the use of DRE special voting aids—our results indicate that each DRE in New York should be allocated to no more than 150 voters, which means replacing each lever machine by 3 DREs. But the acquisition and maintenance cost of this many electronic voting machines would be excessive. In contrast, precinct based, paper ballot optical scan systems use simple, inexpensive marking booths that are the equivalent choke points to DREs. These paper ballot scan systems can be easily and economically configured to eliminate lines.

Voting system choice: timing is everything

Early next year, New York counties will choose either direct recording electronic voting machines (DREs) or paper ballot-scanner systems (PBOS) to replace lever voting machines. How many new voting machines will be needed? The answer to this question is critical for ensuring that each county's voting will go smoothly and that costs will be within reason.

Long lines have occurred during elections with DRE use in California, Florida, Maryland, Mississippi, Ohio, Pennsylvania, Tennessee, Utah and other states[1-9] and have caused some voters to give up and go home, effectively disenfranchising them. It is prohibitive to buy a large number of DREs because of their cost, which makes it likely that a substantial number of voters using DREs will end up in long lines. In contrast, PBOS uses inexpensive marking booths whose numbers can be increased to eliminate lines and long waits.

The New York City Board of Elections recently published a report entitled “An Analysis of the Number of Voters per Voting Machine”[10] which omits several important considerations and contains a number of doubtful assumptions. The result is a serious underestimate of the number of DREs that would be needed to serve the voters of New York City as well as a misunderstanding of relevant aspects of paper ballot-scanner systems.

The New York City Board of Elections report:

- Incorrectly assumes that a maximum of 50% of voters will appear at any election;
- Does not take into account the extra time needed to vote on DREs by persons with disabilities;
- Does not take into consideration the uneven arrival of voters, particularly during peak voting hours, and potential voter traffic jams;
- Does not include the effects of machine and procedural breakdowns.

Properly taking these factors into account substantially decreases the number of voters that could use a voting machine in a day and considerably increases the number of DREs that would have to be purchased and maintained.

In their examination of the use of DREs in the recent Cuyahoga, OH primary, the Election Science Institute carried out a queuing theory analysis of the potential for long lines.[11] Following their approach, we have done our own queuing theory simulation of voting statistics. If we accept the NYC Board of Election report’s figure of 3.25 minutes to vote on a DRE with voter verifiable paper trail—which the report claims will allow 277 voters to use a single machine in a 15-hour voting day—then our study shows that a significant fraction of such elections will have maximum voter waits of over an hour to cast their ballot. This will happen even without the all too common experience of DRE breakdown; it also will occur even if we do not factor in extra time for voters with disabilities. Details of our calculations are given in the Technical Appendix.

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The only way to guarantee short lines is to have a large overcapacity, i.e., to have many more voting systems than would be needed for the average voter flow. This is not practical with DREs because of their high cost. However, New York counties could avoid long lines and save money by choosing paper ballot scanner systems. In the 2004 general election, Lee, MA accommodated 3200 voters on a single paper ballot scanner[12] and Londonderry, NH[13] processed more than 12,000 voters on two scanners in a 13-hour Election Day. Voters mark their ballots in inexpensive marking booths, and there were no lines waiting to mark ballots or to use the scanners in these towns. The number of marking booths can be increased at low cost to avoid any problems of voter traffic congestion and long lines.

Factors not fully considered by the NYC report

Voter turnout will be significantly higher than 50% at some pollsites

The NYC report[10] assumes that a maximum of 50% of registered voters will appear in any election and calculates the number of machines they will order based on this assumption. While the average turnout for New York's 5 counties (New York, Bronx, Kings, Queens and Richmond) was indeed 50% in the 2004 general election, many areas had higher figures.[14,15] So to guarantee efficient access to the polls, it is necessary to consider the peak vote, which could occur in any election precinct.

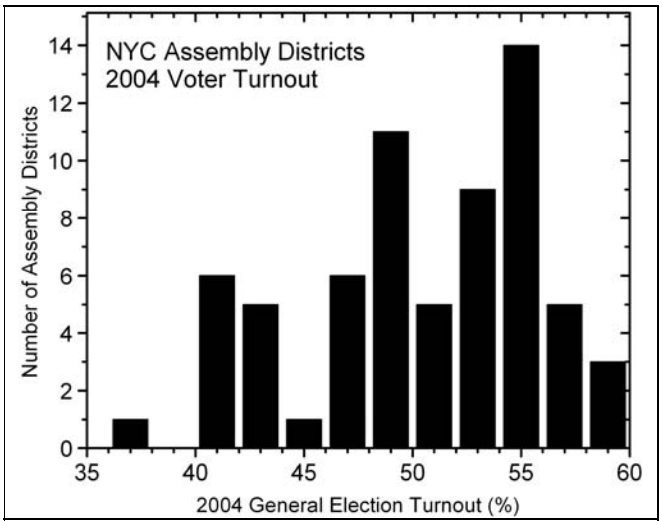

Figure 1. Number of New York City Assembly Districts vs. % turnout in the 2004 general election. Each bar represents how many Assembly Districts had turnout in the range covered by the bar. There were 6 districts that had turnout between 40%-42%, 5 districts had turnout between 42%-44%, 1 district between 44%-46%, etc.

Figure 1 shows the number of Assembly Districts in New York City Counties vs. their votes for President in the 2004 election.[14,15] There are many Assembly Districts where the turnout was well above 50%, indeed approaching 60%. Since these are averages over Assembly Districts, it is apparent that some election precincts within these Districts must have had higher turnouts than 60%. Statewide, over 60% of voters showed up. (The Lee, MA and Londonderry, NH examples cited above had over 80% turnout in 2004.)

The 2004 NY data suggests that an estimate of 75% as an upper bound for voter turnout would be appropriate. This is the same percentage that has been used by NYC to determine how many lever voting machines should be deployed. An underestimate of the number of machines could lead, at least, to serious voter traffic flow problems, long waits and extended Election Days. At worst, it can lead to voter disenfranchisement.

This occurred in the 2004 general election in Florida and Mississippi.[1,2] In Ohio long lines caused voters to give up and leave without voting.[4] There were more long lines in the recent 2006 primary in Cuyahoga County, OH.[11] There were long lines—along with other DRE issues—in the September 2006 problematic primary in Maryland.[3] Insufficient DREs and DRE malfunctions caused more long lines and voter frustration in a number of places in this year's general election on November 7 (e.g. refs. 5-9).

People with special needs will take much longer than 3.25 minutes to vote on DREs

The New York State Board of Elections studied the time needed for persons with special needs to vote with ballot marking devices.[16] This varied from 18 to 45 minutes among the several systems and types of accessibility aids considered. Voting on DREs using accessibility aids would be similar.

The New York State Board of Elections (NYBOE) has hired the American Institutes for Research (AIR)[17] to test voting machines and answers are supposed to be coming in the next few months. Part of their charge is to estimate how many voters would use accessibility aids. This would include, for example, voters with visual, dexterity, or mobility impairments who would use the audio interface or sip and puff controls. It might also include voters who are not comfortable with computers, touchscreens, or the use of English.

A small number of voters with special needs, each taking 30 minutes to vote, would have a profound effect on numbers of voters able to use a DRE on Election Day.

Voters do not arrive at exact intervals

The NYC report allots an average of 3.25 minutes for each voter to use a DRE with voter verifiable paper record. 900 minutes (a 15 hour Election Day) divided by 3.25 is 277. They then assert that a single DRE can accommodate 277 voters, and propose to buy one DRE for each 554 registered voters on the basis of a 50% turnout. However, the NYC report does not properly take into account the effect of fluctuations of voter arrival. It says[10]

> On Election Day, there are "peaks and valleys" of usage by voters depending upon the time of day, the weather, traffic and other variables outside of the control of election staff. Thus there will always be times when voters are waiting, but on the whole, there should be some insurance that waits will not be over long durations throughout the day and that on the whole, voting can be accomplished expeditiously. If we make the assumption that on the whole elections are conducted expeditiously by the survey jurisdictions, than [sic] a maximum that is at, or somewhat higher

than, the average by type of technology should be a reasonable maximum for New York.

These unsupported assumptions are contradicted by DRE delays around the country[1-9] and the mathematics of queuing theory that governs the voting process.

We begin our election simulation by assuming 277 voters per DRE estimated by the NYC report. We take a scenario with heavy voter arrivals from 6am to 8am, 12pm to 1pm and 5pm to 7pm, where voters arrive at double the rate of the rest of the day, and no DREs break down. If the whole-day average is 18.5 arrivals per hour, then the slow periods will have 14 per hour and the fast periods 28 per hour.

There will be many voting locations where only a small number of DREs will be needed,[10] so we have focused—as examples—on pollsites with 1, 2 or 4 DREs. Sites with more DREs will behave proportionally.

With these conditions, over 80% of precincts with 1, 2 or 4 DREs will have voters waiting for more than an hour. Voters will have a maximum wait of more than 1-1/2 hours in 38%, 17% and 3% of precincts where there are 1, 2 or 4 DREs, respectively.

If we have 150 voters per DRE—approximately the ratio of privacy booths to voters specified by statute in NH, scaled by the different Election Day lengths in NH (13 hours) and NY (15 hours)—then there will be few voters waiting more than thirty minutes.

DREs and printers will break, need rebooting, or otherwise cause delays

DRE outages will cause further delays. The present reliability guidelines allow over 9% of voting machines to fail in a 15-hour day,[18] and ESI uses a one-hour average repair/replacement time in some of their calculations.[11]

The bottom line is that even with many more DREs than recommended by the NYC report,[10] queuing statistics guarantee that a substantial number of voting precincts will have voters with very long waits. In order to avoid long lines at DREs for all precincts, it is necessary to have a large excess capacity of voting machines. With DREs, this is not realistic because of their high acquisition and operating costs.

Paper Ballot/Optical Scan: Londonderry, NH and Lee, MA

The following examples of voting by paper ballots and ballot scanners show how PBOS systems can eliminate long voter queues at a minimal cost.

Lee, MA uses PBOS. They had 4,000 registered voters and 3,200 (80%) used 35 privacy booths and one scanner in the 2004 election. The Lee town clerk says that they had no lines at the privacy booths in the 2004 election, whereas they had “long, long lines” when they had previously used 8 lever machines.[12]

Londonderry, NH has 15,029 registered voters and 12,229 (81%) of them voted in the 2004 election.[13] They use PBOS and have two scanners. Each scanner therefore processed about 6114 ballots which is equivalent to 7055 ballots in a 15-hour day. (Note--even if we changed this by a factor of two to accommodate undervote notification in NY, each scanner could still handle 3527 ballots in a day.)

Londonderry has 100 privacy booths, each of which served an average of 122 people in their 13 hour Election Day in 2004, equivalent to 141 voters per NY’s 15 hour Election Day. The town clerk said that there were no lines at the privacy booths. NH requires a privacy booth for every 125 voters. Just in case more voters show up, she has extra cardboard privacy screens that can be placed on tables.

During a heavy election, the Londonderry town clerk estimates that only 10% of booths will be filled much of the day but 90% are occupied during peak times.

Since the scanners only count ballots but do not record the votes, a stopped scanner does not halt the election, unlike DRE failures. If the scanner is down, the ballots are placed in a special compartment in the ballot bin and scanned later. The Londonderry town clerk has supervised 25-50 elections over the last 7 years and has experienced only one scanner breakdown. A replacement scanner was brought over and put into service in less than an hour.

These examples show the kind of overcapacity that works. It is easy to achieve with PBOS since the privacy booths cost about $150 each. In reality, it would take more than 100 DREs in Londonderry, NH or 35 DREs in Lee, MA to achieve the same ease of use since 1) DREs have a much greater breakdown rate than scanners and 2) voters with disabilities would take up a lot of DRE capacity. In contrast, ballot-marking machines that go with PBOS are separate devices that do not affect flow of voters in other privacy booths or the operation of the scanners.

How many voting machines do we need?

Direct Recording Electronic Voting Machines (DREs)

The NYC report suggests that each DRE could serve 277 voters on Election Day. Assuming a 50% turnout, they conclude that one DRE should be purchased for each 554 registered voters, which is not too different from what the report says is an average of 1 lever machine for 600 registered voters. In other words, they recommend replacing each lever machine by a single DRE.[10]

Our study shows that 277 voters per machine is unrealistic (given the NYC voting time figure of 3.25 minutes) and will lead to very long waiting times in some election districts. We believe a realistic ratio that keeps lines down everywhere would be more like 150 voters per DRE. Trying to serve even this number of voters with one DRE may prove problematic because of DRE outages and long voting times for persons with special needs. Taking 150 voters per DRE and a possible 75% turnout implies a DRE for each 200 registered voters. The replacement ratio then becomes 3 DREs for each lever machine.

The time for each person to vote (3.25 minutes) in our calculation was taken from the NYC report and could change when the “usability study” is completed by AIR. Any figures then obtained can—and should—be used for a queuing analysis similar to what we have done here.

Paper Ballot, Ballot Marker, Optical Scan Systems (PBOS)

The examples above from Londonderry, NH and Lee, MA demonstrate that a large overcapacity, i.e. one privacy booth for 125 voters (Londonderry, NH) or per 90 voters (Lee, MA) essentially eliminated the experience of voting bottlenecks. This is a simple and inexpensive solution which would improve the voting experience in New York, as it did when Lee, MA went from 8 lever machines for 3200 people to 35 privacy booths.

If we scale the NH number from a 13 hour day to a 15 hour day, then one needs approximately one privacy booth for each 150 voters who show up. Assuming a 75% turnout, we therefore need a privacy booth for each 200 registered voters. As is done by the Londonderry, NH town clerk, it is a good idea to have a number of additional folding cardboard privacy screens that can be placed on tables in case more voters come.

As mentioned above, ballot marking devices that would be used by voters with disabilities will not affect the voting process for other voters. With this option, election commissioners have to determine how many disabled voters will vote in their election district and buy enough ballot marking devices to serve their disabled constituents. Commissioners should be aware that as voters with disabilities become more familiar with new voting technology, their attendance at polling sites will increase..

Acquisition Costs

A simple calculation for a pollsite with 2,000 registered voters shows how PBOS could save $45,500 in acquisition costs compared to buying DREs.

According to our figures above, this pollsite would require 10 DREs at approximately $8,000 each or $80,000.

The same pollsite could be served by a single optical scanner ($5,500) and 10 marking booths ($150 each, $1,500). It would also need ballot marking devices for the disabled.

We now calculate the number of ballot marking devices needed. Let us assume a 75% turnout (1,500 voters), that 5% of those voters (75) need special access and that each takes 30 minutes. 900 minutes divided by 30 minutes is 30. There will be a similar queuing problem for the ballot markers as there was for the DREs, so that number should probably be decreased by a factor of 2 to 15. Then the district should buy 5 ballot marking devices at about $5,500 each which comes to a total of $27,500.

Thus the total is $5,500 + $1,500 + $27,500 = $34,500 for PBOS and ballot marking device acquisition costs, $45,500 less than the figure for DREs.

Conclusions: DREs will cause long lines; PBOS can eliminate lines

The use of DREs has created long lines in many constituencies around the country. The nature of voter arrival statistics is such that there may be a large variability in the waiting times for different voting locations as governed by the mathematics of queuing theory. The only way to avoid long waiting times for voters is to have a large overcapacity, i.e. many DREs or many marking booths for use with PBOS. This is only economically possible with PBOS, as DREs ($8,000) represent the equivalent choke point in the voting process as the marking booth ($150).

DRE outages (10% in recent experience) and long voting times for persons with special needs represent further potential serious slowdowns on Election Day.

We believe that 30 minutes should be the maximum waiting time for voters. Many of them may be taking time off work, have to manage accompanying children or have medical conditions that make it difficult for them to stay at the polling place for extended times. Every precinct that follows the advice of the New York City Report to allocate one DRE for each 277 actual voters will exceed that standard. Most will have maximum wait times of at least one hour, and a significant number will have wait times greater than 1-1/2 hours.

Recent experience shows that such a result produces frustration or hardship for voters and many will leave rather than wait in long times. The analysis in the Technical Appendix shows that numerous polling places allocating 1 DRE for each 277 voters will also have substantial overtimes, creating long workdays for election workers. 150 voters per DRE might work reasonably well, although the picture is clouded by DRE breakdown and use by disabled voters.

These numbers are similar to those in a story about a voting precinct with 2 DREs in Nashville, TN. Elections went well with 214 voters (107 per DRE) but had long lines with 527 voters (263 per DRE).[5]

The correct and smooth functioning of elections is fundamental to democracy. Everything compatible with election integrity should be done to make the process voter-friendly—overlong waits are unacceptable. If a city has hundreds of precincts and ten of them have multi-hour waiting lines or Election Day delays which force voters to continue well beyond midnight—as happened in Ohio in 2004—voters will blame election commissioners and other government officials for not having enough machines. People will leave without voting; this amounts to disenfranchisement of those voters. There will be angry charges that the election has been compromised or manipulated.

The uneven flow of voters on Election Day means that the only way to guarantee equal voter access in terms of the time it take to vote is to have a sizable overcapacity in every district. Both acquisition and operating costs make that economically prohibitive with DREs. More machines mean higher operating costs as well as higher acquisition costs. To provide for reasonable waiting times, it will be necessary to have three times as many DREs as the NYC Board of Election suggests. In contrast, it is eminently feasible to have negligible lines for PBOS systems, because it is possible to supply a large number of inexpensive privacy booths for marking ballots.

Technical Appendix: Queuing Theory, Voter/Voting Machine Ratios, and Long Lines

"Queuing theory…is the mathematical study of waiting lines."[19] We are concerned here with lines of people waiting to use voting machines. Queuing theory in this case uses voter arrival rate, the number of available machines, the time for each voter to vote and the machine breakdown rate to predict the probability of forming long lines during Election Day and overtime at the end of the day. We have applied this approach to a few simple scenarios to show that the numbers of DRE machines proposed by the NYC report would lead to long lines, many with delays of one to two hours or even longer.

The NYC report starts its calculations with the premise that each voter will take approximately 3.25 minutes to vote on a DRE with a voter verifiable paper trail. They then divide a 15-hour voting day (900 minutes) by 3.25 minutes and conclude that 277 people can all vote on a single machine in one day, or, conversely, that it is only necessary to buy one DRE for every 277 actual voters.

This might be OK if people were to arrive precisely every 3.25 minutes like clockwork. In reality, they come to the polls randomly according to a Poisson process with an exponential distribution of intervals between arrivals.[20] Sometimes they drop in more frequently than the average rate and have to wait. Sometimes they show up more slowly and machines sit idle, wasting time that cannot be made up and that inevitably lead to lines or overtime at the end of the day.

More voters come early in the morning, at lunch or after work and during the dinner hour than during the rest of the day. The NYC estimate does not take this into account. Neither does it factor into its analysis DRE outages that occur with a 10% probability and take an average of one hour to repair. In addition, it does not include the fact that persons with disabilities will use special functionalities of the DREs and take extra time to vote.

Our calculations were based on the use of 1, 2 or 4 DREs, as there will be many pollsites with a small number of DREs.[10] Larger sites would operate proportionately.

Queuing calculations: methodology

Based on references 11 and 21, we wrote a computer program to simulate voting during a 15-hour Election Day, from 6am to 9pm. From 6am to 8am, 12pm to 1pm, and 5pm to 7pm the arrival rate was double that for the rest of the day. We carried out calculations for 1, 2 or 4 DREs in a pollsite under the following conditions.

1. An average of 277 voters per DRE as estimated by the NYC Board of Elections;
2. An average of 150 voters per DRE, approximately the equivalent to the requirement of 125 privacy booths for PBOS in NH for a 13-hour Election Day.
3. The effect of DRE outages for 150 voters per DRE;
4. The effect of disabled voters and voters with other special needs who take 30 minutes each to vote.

Calculations for DREs without factoring in outages or voters with special needs

Two exponential distribution sets of random arrival intervals are generated, one for most of the day and another for the higher rate between 6-8am, 12-1pm and 5-7pm. Each voter is then assigned to the machine which has finished (or will finish) earliest with previous voters. If the machine is available, then that machine is occupied for 3.25 minutes. If the machine is not yet free, the voter waits for it to be available and then takes 3.25 minutes to finish. In some cases the voter may arrive well after one of the machines has been free.

We calculated the case for 150 voters per DRE first without, and later with, DRE outages.

Maximum waiting times and waiting time for the last voter (overtime) are then extracted from the results. Each simulation for 1, 2 or 4 DREs is repeated 10,000 times to get a statistical distribution of maximum waiting times and overtimes.

Figure A1 shows a waiting pattern for 2 DREs. Taking the NYC suggested number of 277 voters per DRE in the day, long lines develop during and following higher arrival rates at 6-8am, 12-1pm and 5-7pm. In this example, voters around 8am are waiting 80 minutes, and there is a 20 minute overtime at the end of the day.

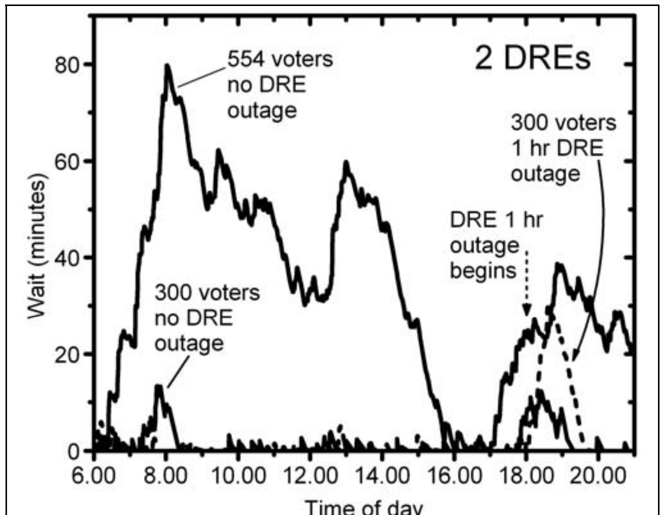

Figure A1. Typical waiting patterns for a 2 DRE precinct. In the scenario for the NYC figure of 277 voters per DRE (554 total), voters arrive every 1 minute during 6-8am, 12-1pm and 5-7pm, and 1 every 2 minutes during the rest of the day. For 150 voters, the corresponding figures are 1.9 and 3.8 minutes. Voter waiting times are shown for: a total of 554 voters (277 voters per DRE); 300 voters (150 voters per DRE); and 300 voters where one DRE has a 1-hour outage. The 554 voter example has extensive waiting times set off by the high turnouts during morning, lunch and evening.

For 150 voters per DRE (total 300), the longest waiting time is about 10 minutes. The DRE outage occurring around 6pm produces a 30 minute maximum wait.

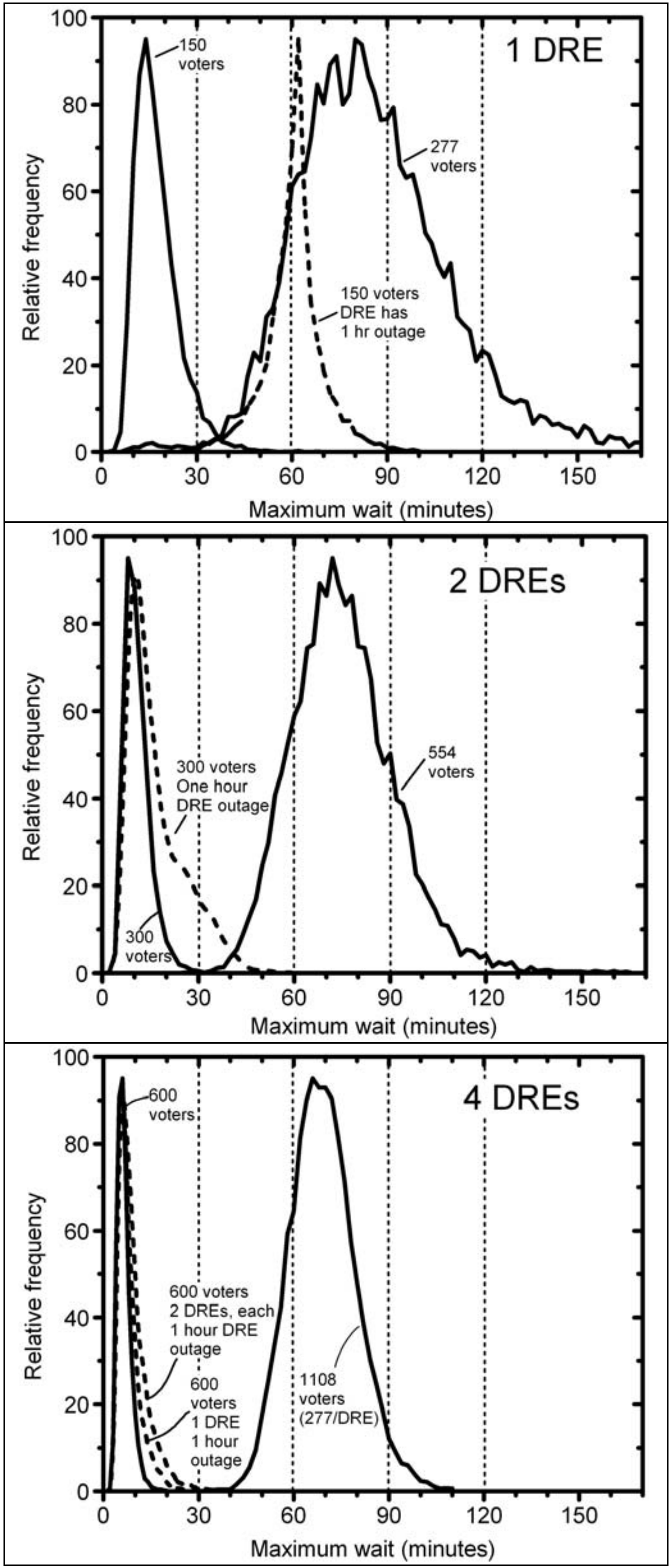

Figure A2. Distribution of maximum waiting times for voting precincts with 1, 2 or 4 DREs. Curves are shown for an average of 277 voters per DRE and for 150 voters per DRE. The results for 150 voters per DRE with 1-hour DRE outages are also displayed. The curves are histograms derived from runs of 10,000 simulations, and heights have been normalized in order to make comparison easier.

Figure A2 shows a distribution of maximum waiting times for 1, 2 and 4 DREs for 277 voters per DRE and 150 voters per DRE. 277 voters per DRE results in long waits averaging 70-80 minutes. The distribution for 1 DRE is wide and there will be a significant number of people waiting more than 2 hours. The fraction of the time voters will have to wait more than 30, 60, 90 or 120 minutes is the integral of the curve to the right of each particular time.

Table A1 shows the results for precincts with 1, 2 and 4 DREs respectively for an average 277 voters per machine, no DRE outages. Table A2 shows the corresponding results for 150 voters per DRE.

Table A1. Maximum wait time and overtime for 277 voters per DRE, 15 hr Election Day from 7 am to 10 pm. Arrival rate twice as high from 6 am to 8 am, 12 pm to 1 pm, 5 pm to 7 pm, as rest of day. No DRE outages and no voting with disability aids.

Maximum wait						
# DREs	Avge # of voters	**Avge max wait (min)**	>30 min max wait	>60 min max wait	>90 min max wait	>120 min max wait
1	277	85.9	100.0%	88.3%	38.1%	8.4%
2	554	75.4	100.0%	83.6%	16.9%	1.2%
4	1108	69.2	100.0%	81.1%	3.3%	0.0%
Overtime						
		Avge over-time (min)	>30 min over-time	>60 min over-time	>90 minutes over-time	>120 min over-time
1	277	45.1	64.8%	29.1%	8.6%	2.0%
2	554	37.0	60.0%	14.2%	1.5%	0.5%
4	1108	33.2	56.8%	4.6%	0.4%	0.0%

Table A2. Maximum wait time and overtime for 150 voters per DRE, same conditions as Table A1.

Maximum wait						
# DREs	Avge # of voters	**Avge max wait (min)**	>30 min max wait	>60 min max wait	>90 min max wait	>120 min max wait
1	150	17.3	5.3%	0.0%	0.0%	0.0%
2	300	10.9	0.1%	0.0%	0.0%	0.0%
4	600	6.6	0%	0%	0%	0%
Overtime						
		Avge over-time (min)	>30 min over-time	>60 min over-time	>90 minutes over-time	>120 min over-time
1	150	1.1	0%	0%	0%	0%
2	300	0.3	0%	0%	0%	0%
4	600	0.1	0%	0%	0%	0%

Figure A2 also has plots showing the effect of DRE failure in polling places with 150 voters per DRE. Voter waits in a precinct with one DRE are seriously affected with maximum waits around one hour, as would be expected. The effects in 2-DRE or 4-DRE precincts are not a problem in this case. However, given the 10% failure rate of DREs, this result says that there should be no precincts with a single DRE, even if there are 150 voters or fewer.

These figures match well the experience of a 2-DRE voting precinct in Nashville, TN. An election with 214 voters (107 per DRE) went well. The 2006 general election had 527 voters (263 per DRE), 4-hour waits and an overtime of 5-1/2 hours. [5]

Voters with disabilities

The NY State Board of Elections tested ballot marking devices for the disabled and found that it took 18-45 minutes to vote. We have done voting day simulations assuming the average value 30 minutes for disabled voters and 3.25 minutes for non-disabled voters on DREs that are scheduled to serve about 150 voters each, the figure we estimate from PBOS usage. Even with only 150 voters per DRE, substantial delays will occur with a relatively small number of disabled voters.

Figure A3 is an example of waiting times for 4 DREs with a total of 20 disabled voters (out of 600 voters total) with their arrival times—determined by random numbers—shown in the graph. As discussed above, 4 DREs can handle 600 voters with essentially no waiting times. Figure A3 has a sizable number of disabled voters appearing at busy times. About 7 come between 5pm and 7pm, which causes a large accumulated delay.

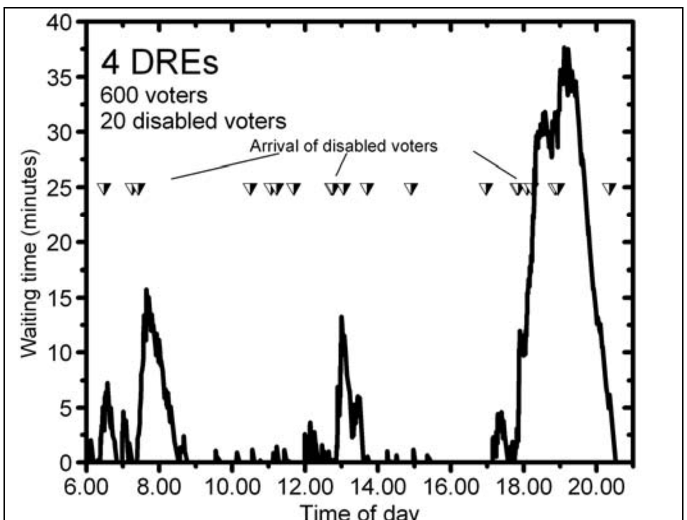

Figure A3. Waiting times for 4 DREs with 20 disabled voters arriving randomly throughout the day. The downward pointing triangles indicate the disabled voter arrival times.

Figure A4 shows distributions of maximum waits for 1, 2 and 4 DREs with 0, 2, 5 and 10 disabled voters per DRE (150 total voters per DRE). The distributions were derived by running 10,000 simulated elections for each case. The adverse effects decrease for more DREs. Increasing the number of DREs tends to smooth out the perturbations in produced by 30-minute voting periods.

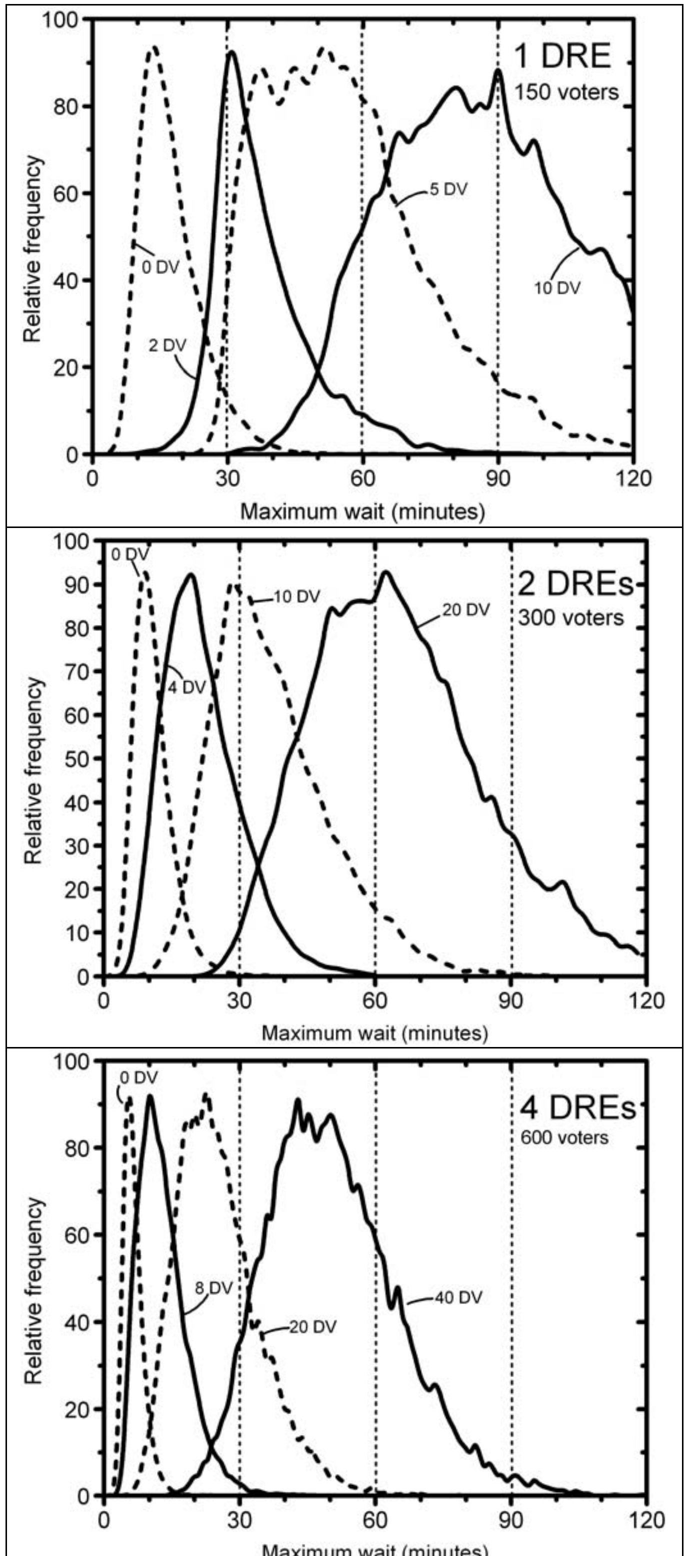

Figure A4. Distributions of maximum waiting times for DREs with disabled voters. The DREs average a total of 150 voters each. Simulations are shown for 0 disabled voters (0 DV), 2 DV, 5 DV and 10 DV per DRE. 10 DV/DRE is only 6.7% of the total 150 and 3.6% of the 277 voters per DRE suggested by the NYC Board of Elections. It is apparent that, for 1 DRE, a substantial number of voters will have waits of longer than 1 hour if either 5 or 10 disabled voters come to vote. 5 or 10 disabled voters per DRE will also cause long waits for a pollsite with 2 or 4 DREs.

Table A3 shows the maximum waiting times as a function of voters with disabilities for precincts of 1, 2 or 4 DREs. With only 2 such voters, 76% of the 1-DRE precinct will have greater than 30 minute maximum waits. With 10 voters with disabilities per DRE, over 90% of pollsites would have greater than 30 minute waits in every election. 1-, 2- and 4-DRE districts will have greater than 1 hour maximum waits 91%, 58% and 24%, respectively.

10 voters with disabilities per DRE is only 6.7% of the 150 voter figure or 3.6% of the 277 voters per DRE suggested by the New York City Board of Elections.

Table A3. Maximum waiting time for varying numbers of disabled voters (DV). 150 total voters per day per DRE with 0 DV/DRE, 2 DV/DRE, 5 DV/DRE and 10 DV/DRE.

	1 DRE	2 DREs	4 DREs
0 DV/DRE	**0 DV**	**0 DV**	**0 DV**
> 30 min	5.2%	0.2%	0.0%
> 60 min	0.0%	0.0%	0.0%
> 90 min	0.0%	0.0%	0.0%
> 120 min	0.0%	0.0%	0.0%
2 DV/DRE	**2 DV**	**4 DV**	**8 DV**
> 30 min	76.4%	16.5%	0.9%
> 60 min	5.1%	0.1%	0.0%
> 90 min	0.1%	0.0%	0.0%
> 120 min	0.0%	0.0%	0.0%
5 DV/DRE	**5 DV**	**10 DV**	**20 DV**
> 30 min	98.2%	67.1%	27.1%
> 60 min	37.6%	6.1%	0.2%
> 90 min	5.8%	0.1%	0.0%
> 120 min	0.6%	0.0%	0.0%
10 DV/DRE	**10 DV**	**20 DV**	**40 DV**
> 30 min	100.0%	99.3%	94.5%
> 60 min	90.6%	57.9%	24.1%
> 90 min	47.7%	12.3%	1.2%
> 120 min	15.2%	1.3%	0.0%

Figure A5 is a parametric plot of the fraction of elections with maximum waits of 30 or 60 minutes as a function of voters per DRE for pollsites with 1, 2 or 4 DREs. For example, 1% of the pollsites with 2 DRE will have waits of over 60 minutes if there are more than about 220 voters allocated to vote on that DRE.

Real conditions can vary, but we can use this plot and information derived above to develop a criterion for specifying the number of voters per DRE. We suggest a cutoff of 0.1% (1 in 1000 elections) for maximum waits over 30 minutes. This would allow 110 voters in 1 DRE polling places, 150 voters per DRE (280 voters) in 2 DRE pollsites, and 170 voters per DRE (680 voters) in 4 DRE pollsites. A reasonable average value drawn from these data would be 150 voters per DRE in all pollsites.

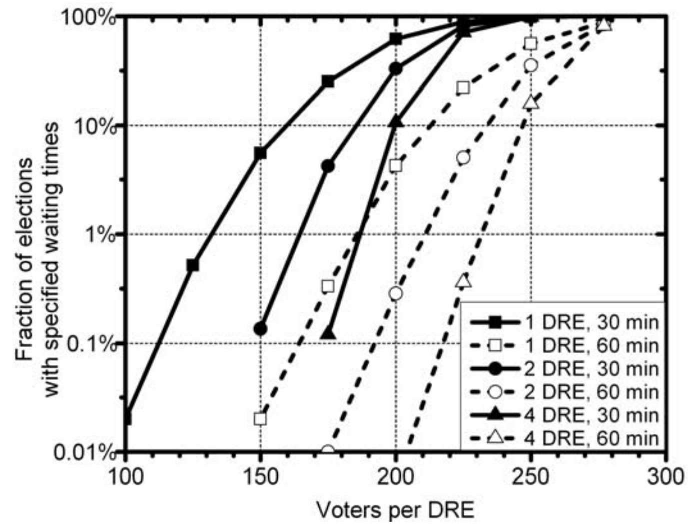

Figure A5. Fraction of elections with maximum waits of 30 minutes or 60 minutes vs. voters per DRE for 1, 2 or 4 DREs. To keep maximum waits below 30 minutes in 0.1% (1/1000) of elections, it would be necessary to have about 110 voters for 1 DRE, 140 voters per DRE with 2 DREs, and 170 voters per DRE with 4 DREs. 150 voters per DRE would be a good starting value.

0.1% may seem very conservative, but there are over 7600 polling places in New York.[22] Thus failures of 1 per 1000 polling places could have 8 pollsites with maximum waits over 30 minutes.

If we assume a 75% maximum turnout, then 150 actual voters per DRE translates to one DRE for every 200 registered voters, i.e. 3 DREs for every lever machine. This is not too different from the requirement of 1 DRE for every 175 registered voters in Ohio which is supposed to be enforced in 2013.[4] Of course New York's full face DREs at $8,000 each will be much more expensive than DREs for Ohio.

This calculation of voters per DRE does not take into account DRE outages or voters with special needs who will take a long time to vote. This will tend to increase the number of DREs needed.

Appendix Conclusion

Queuing theory is an important tool in the understanding of voting system use. Because there are busy and slack periods during the voting day, and because people do not arrive at a uniform rate, the number of people that can vote on a given system is far less than is calculated by simply dividing the total Election Day time by the time for an individual to vote. Any voting frequency figures obtained by the NY State Board of Elections should be analyzed in the framework of queuing theory.

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